New Zealand Mathematical Olympiad Committee

Camp Selection Problems 2017 — Instructions

Solutions due date: 29th September 2017

These problems will be used by the NZMOC to select students for its International Mathematical Olympiad Training Camp, to be held in Auckland over 7–13 January 2018. Only students who attend this camp are eligible for selection to represent New Zealand at the 2018 International Mathematical Olympiad (IMO), to be held in Romania in July 2018. We have been fortunate in obtaining some sponsorship for the camp so have been able to keep the cost to $650, which covers all expenses for the camp including travel for those from outside Auckland.

At the camp a squad of 10–12 students will be chosen for further training. The New Zealand team for the 2018 IMO will be chosen from this squad. Those selected for the camp will also be invited to sit Round one of the British Mathematical Olympiad (BMO1), to be held on 2nd December 2017.

Students will be selected into two groups for the camp: juniors and seniors.

- If you are currently in year 12, **or** you have been a member of the NZIMO training squad, then you will be considered only as a senior.
- Otherwise, you will be considered as either a junior or a senior.

Since the problems and the participants vary from year to year it is hard to be precise about the selection criteria. However, as a rough guide, if you solve five or more of the problems completely then you will be in the running for selection as a junior. Obviously, the criteria for senior selection are somewhat higher.

General instructions:

- All solutions must be entirely your own work.
- You may not use a calculator, computer or the internet (except as a reference for e.g., definitions) to assist you in solving the problems.
- Although some problems seem to require only a numerical answer, in order to receive full credit for the problem a complete justification must be provided. In fact, an answer alone will be worth at most 20% of the credit for a problem.
- We do not expect many, if any, perfect submissions. So, please submit all the solutions and partial solutions that you can find.
- Please address **all** queries about the problems to Chris Tuffley, c.tuffley@massey.ac.nz.

Students submitting solutions should be intending to remain in school in 2018 and should also hold New Zealand Passports or have New Zealand Resident status. To be eligible for the 2018 IMO you must have been born on or after 1st July 1998, and must not be formally enrolled in a University or similar institution prior to the IMO. If you are uncertain about your eligibility please contact Chris Tuffley (c.tuffley@massey.ac.nz) **before** submitting solutions.

Your solutions, together with a completed Registration Form (overleaf), should be sent to

> Dr Christopher Tuffley, Institute of Fundamental Sciences, Massey University Manawatū, Private Bag 11222, Palmerston North 4442, New Zealand

arriving no later than 29th September 2017. Please complete the registration form **carefully** and **legibly**, in particular your contact details. We regret that we are unable to accept electronic submissions. You will be notified whether or not you have been selected for the camp by 31st October 2017.

September 2017

Registration Form
NZMOC Camp Selection Problems 2017

This form is a fillable PDF. You can type your answers into the form and then print and sign it to send in with your answers, or you can print it and then fill it in by hand.

Name: ______________________________

Gender: ______________ **School year level in 2018:** ________

Home address: ______________________________

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Email address: ______________________________

Home phone number: ______________ **Cell:** ______________

School: ______________________________

School address: ______________________________

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Principal: ______________________________

HOD Mathematics: ______________________________

Do you intend to take part in the camp selection problems for any other Olympiad camp?	yes	no
If so, and if selected, which camp would you prefer to attend? ______________		
Have you put your name forward for a Science camp or any other camp in January?	yes	no
Are there any criminal charges, or pending criminal charges against you?	yes	no

Some conditions are attached to camp selection. You must be:

- Born on or after 1st July 1998
- Studying in 2018 at a recognised secondary school in NZ
- Available in July 2018 to represent NZ overseas as part of the NZIMO team if selected.
- A NZ citizen or hold NZ resident status.

Declaration: I satisfy these requirements, have worked on the questions without assistance from anyone else, and have read, understood and followed the instructions for the January camp selection problems. I agree to being contacted through the email address I have supplied.

Signature: ______________________________ Date: ______________

Attach this registration form to your solutions, and send them to

> Dr Christopher Tuffley, Institute of Fundamental Sciences, Massey University Manawatū, Private Bag 11222, Palmerston North 4442, New Zealand,

arriving no later than 29th September 2017.

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Notes

- If any clarification is required, please contact Chris Tuffley (c.tuffley@massey.ac.nz).
- There are nine problems and you should attempt to find solutions for as many as you can. Solutions (i.e., answers with justifications) and not just answers are required for all problems, even if they are phrased in a way that only asks for an answer.
- These are selection problems only: you will not receive a 'score', only an indication of whether you were selected.
- For further information, see the general instructions on the registration form that accompanies these problems.

Problems

1. Alice has five real numbers $a < b < c < d < e$. She takes the sum of each pair of numbers and writes down the ten sums. The three smallest sums are 32, 36 and 37, while the two largest sums are 48 and 51. Determine e.

2. Let $ABCD$ be a parallelogram with an acute angle at A. Let G be the point on the line AB, distinct from B, such that $CG = CB$. Let H be the point on the line BC, distinct from B, such that $AB = AH$.

   Prove that triangle DGH is isosceles.

3. Find all prime numbers p such that $16p + 1$ is a perfect cube.

4. Ross wants to play solitaire with his deck of n playing cards, but he's discovered that the deck is "boxed": some cards are face up, and others are face down. He wants to turn them all face down again, by repeatedly choosing a block of consecutive cards, removing the block from the deck, turning it over, and replacing it back in the deck at the same point. What is the smallest number of such steps Ross needs in order to guarantee that he can turn all the cards face down again, regardless of how they start out?

5. Find all pairs (m, n) of positive integers such that the $m \times n$ grid contains exactly 225 rectangles whose side lengths are odd and whose edges lie on the lines of the grid.

6. Let $ABCD$ be a quadrilateral. The circumcircle of the triangle ABC intersects the sides CD and DA in the points P and Q respectively, while the circumcircle of CDA intersects the sides AB and BC in the points R and S. The lines BP and BQ intersect the line RS in the points M and N respectively. Prove that the points M, N, P and Q lie on the same circle.

7. Let a, b, c, d, e be distinct positive integers such that

   $$a^4 + b^4 = c^4 + d^4 = e^5.$$

   Show that $ac + bd$ is composite.

8. Find all possible real values for a, b and c such that

   (a) $a + b + c = 51$,

   (b) $abc = 4000$,

   (c) $0 < a \leq 10$ and $c \geq 25$.

9. Let k and n be positive integers, with $k \leq n$. A certain class has n students, and among any k of them there is always one that is friends with the other $k - 1$. Find all values of k and n for which there must necessarily be a student who is friends with everyone else in the class.

September 2017

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