New Zealand Mathematical Olympiad Committee

NZMO Round One 2021 — Instructions

Submissions due date: 30^{th} July

The **New Zealand Mathematical Olympiad** (NZMO) consists of two rounds:

Round One (the NZMO1): A take home exam (the following set of 8 problems). Solutions are to be submitted by 30^{th} *July*.

Round Two (the NZMO2): A three hour supervised exam in September.

Awards for the NZMO (gold/silver/bronze/honourable mention) will be made based on the results of the NZMO2. Scores will not be announced for either round. Participants in the NZMO1 will only receive an indication of whether they have been invited to participate in the NZMO2; participants in the NZMO2 will just be told the level of their award, if any.

In addition, the results of both rounds of the NZMO will be used to select about 25 students to participate in the NZMOC training camp, to be held at the University of Auckland in January 2022. Only students who are New Zealand citizens or permanent residents, and who will still be enrolled in intermediate or high school in 2022, are eligible for NZMOC training camp selection. The training camp is the first step in the selection process to choose the team of six students to represent New Zealand at the 2022 International Mathematical Olympiad, (IMO). Only students who attend this camp are eligible for team selection.

General instructions:

- The NZMO is an individual competition. Participants must work on the problems entirely on their own, without assistance from anyone else or any kind of software.
- Electronic devices may not be used to assist in solving the problems. This includes but is not limited to calculators, computers, tablets, smart phones and smart watches.
- The internet may not be used, except as a reference for looking up definitions.
- Although some problems may seem to require only a numerical answer, in order to receive full credit for the problem a complete justification must be provided.
- Submitting rough working and/or partial solutions cannot negatively affect the score awarded. We encourage you to submit all rough work and any partial solutions that you can find.
- Please address **all** queries about the problems to Dr Ross Atkins, info.nzmoc@gmail.com

Participation in the NZMO is open to any student enrolled in the New Zealand education system, at secondary school level or below. If you are uncertain about your eligibility please contact Dr Ross Atkins (info.nzmoc@gmail.com) **before** submitting solutions.

The NZMO1 submission url is: https://www.mathsolympiad.org.nz/nzmo1_submission. If you are having trouble loading the submission form, please try logging out of your google account, or opening the form in an incognito window. All your solutions and partial solutions should be submitted as a **single document** in PDF format. Please complete the submission form **carefully**, especially your contact details. If you have any difficulties with the submission form, contact Dr Ross Atkins (info.nzmoc@gmail.com).

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NZMO Round One 2021 — Problems

Submissions due date: 30^{th} July

- There are 8 problems.
- Read and follow the "General instructions" accompanying these problems.
- If any clarification is required, please contact Dr Ross Atkins (info.nzmoc@gmail.com).

Problems

1. A school offers three subjects: Mathematics, Art and Science. At least 80% of students study both Mathematics and Art. At least 80% of students study both Mathematics and Science. Prove that at least 80% of students who study both Art and Science, also study Mathematics.

2. Let $ABCD$ be a trapezium such that $AB \parallel CD$. Let E be the intersection of diagonals AC and BD. Suppose that $AB = BE$ and $AC = DE$. Prove that the internal angle bisector of $\angle BAC$ is perpendicular to AD.

3. In a sequence of numbers, a term is called *golden* if it is divisible by the term immediately before it. What is the maximum possible number of golden terms in a permutation of $1, 2, 3, \ldots, 2021$?

4. Find all triples (x, p, n) of non-negative integers such that p is prime and
$$2x(x + 5) = p^n + 3(x - 1).$$

5. Let ABC be an isosceles triangle with $AB = AC$. Point D lies on side AC such that BD is the angle bisector of $\angle ABC$. Point E lies on side BC between B and C such that $BE = CD$. Prove that DE is parallel to AB.

6. Is it possible to place a positive integer in every cell of a 10×10 array in such a way that both the following conditions are satisfied?
   - Each number (not in the top row) is a proper divisor of the number immediately above.
   - Each row consists of 10 consecutive positive integers (but not necessarily in order).

7. Let a, b, c, d be integers such that $a > b > c > d \geq -2021$ and
$$\frac{a+b}{b+c} = \frac{c+d}{d+a}$$
(and $b + c \neq 0 \neq d + a$). What is the maximum possible value of ac?

8. Two cells in a 20×20 board are *adjacent* if they have a common edge (a cell is not considered adjacent to itself). What is the maximum number of cells that can be marked in a 20×20 board such that every cell is adjacent to at most one marked cell?