New Zealand Mathematical Olympiad Committee

Maths Workshop

December 2021

Problems

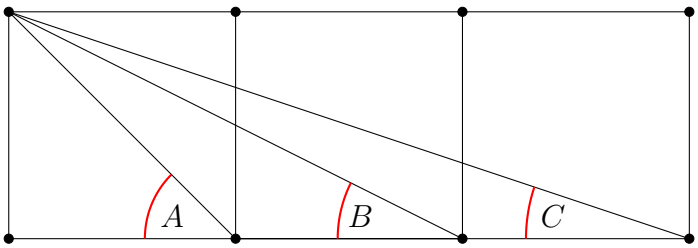
1. On an island there are knights and knaves. Knights always tell the truth and Knaves always lie. Josie finds two islanders and asks them the following question:

   "*Is either of you a Knight?*"

   When one of them answered, Josie could deduce what each of them was. Was the answer "*Yes*" or "*No*"?

2. Solve the inequality
$$\left|\frac{x}{x^2-4}\right| \geq \frac{1}{3}$$

3. Below are three adjacent squares. Determine the value of $\angle A + \angle B + \angle C$.



4. Eleven non-zero digits are written around a circle. Any two adjacent digits can be read clockwise as a two-digit number. Is it possible that the product of these eleven two-digit numbers is a perfect square?

5. Let $\phi(n)$ denote the number of positive integers less than n, which are relatively prime to n. How many positive integers n exist such that $\phi(n) = 128$?

6. Given are 12 distinct marbles, 4 of which are red. In how many ways can the marbles be arranged in a line so that no pair of the 4 red marbles are adjacent?