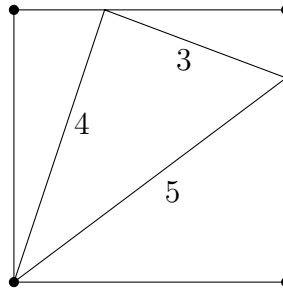


## Problems

1. A triangle with sidelengths 3, 4 and 5 is inscribed in a square as shown. What is the area of the square?



2. A regular dodecahedron is orthogonally projected onto a plane. The image is an  $n$ -sided polygon. What is the smallest possible value of  $n$ .
3. Find all real numbers  $x$  and  $y$  such that

$$x^2 + y^2 = 1 \quad \text{and} \quad x^3 + y^3 = 1.$$

4. Prove that the fraction  $\frac{21n + 4}{14n + 3}$  is in simplest terms for every positive integer  $n$ .
5. Twenty-seven identical white cubes are assembled into a single cube, the outside of which is painted black. The cube is then disassembled and the smaller cubes thoroughly shuffled in a bag. A blindfolded person (who cannot feel the paint) randomly reassembles the pieces into a cube. What is the probability that the outside of this cube is completely black?
6. In  $\triangle ABC$ , side  $AB = 20$ ,  $AC = 11$  and  $BC = 13$ . Find the diameter of the semicircle inscribed in  $ABC$ , whose diameter lies on  $AB$ , and that is tangent to  $AC$  and  $BC$ .