



Problems

1. Determine the value of the following series.

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots + 99 - 100$$

2. Consider two random 3 digit numbers chosen uniformly at random (between 100 and 999 inclusive). What is the probability that there are no *carries* when these two numbers are added together?
3. Find all prime numbers p such that $5p + 2$ is a square number.
4. Let ABC be any triangle and let D , E and F be the midpoints of AB , BC and CA . Let X be the point on BC such that AX is perpendicular to BC . Prove that X lies on the circumcircle of DEF .
5. Does there exist a (base 10) 23-digit multiple of 2^{23} , written exclusively with the digits 2 and 3?
6. A triangle has sides 10, 17, and 21. A square is inscribed in the triangle. One side of the square lies on the longest side of the triangle. The other two vertices of the square touch the two shorter sides of the triangle. What is the length of the side of the square?